

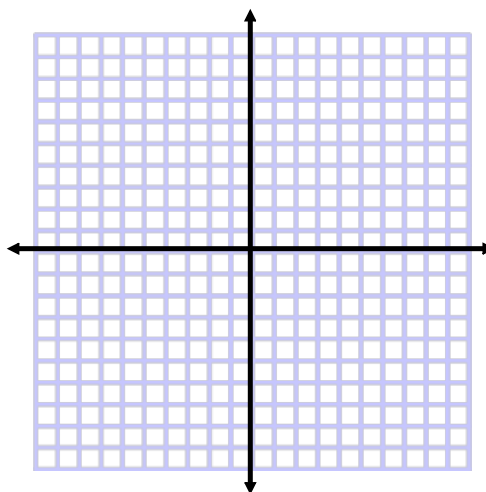
Section 7.4 - Reciprocal Functions

Part A: Reciprocal Functions of Linear Functions

Compare the Graphs of a Function and Its Reciprocal

Sketch the graphs of $y = f(x)$ and its reciprocal function $y = \frac{1}{f(x)}$, where $f(x) = x$. Examine how the functions are related.

| x | y=x | y=1/x |
|-------|-----|-------|
| -10 | | |
| -1/10 | | |
| 0 | | |
| 1/10 | | |
| 10 | | |



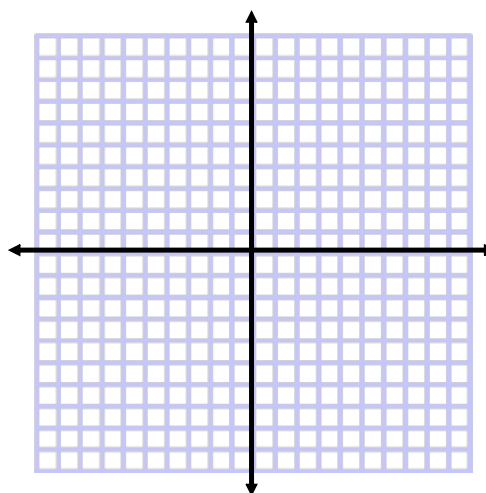
| Characteristic | y=x | y=1/x |
|------------------|-----|-------|
| Domain | | |
| Range | | |
| End Behaviour | | |
| Behaviour at x=0 | | |
| Invariant Points | | |

Example:

Create a table of values and sketch the graphs of $y = f(x)$ and its

reciprocal $y = \frac{1}{f(x)}$, where $f(x) = -x$.

| x | $y = -x$ | $y = -1/x$ |
|-------|----------|------------|
| -10 | | |
| -1/10 | | |
| 0 | | |
| 1/10 | | |
| 10 | | |



Define each of the following.

Asymptote:

Vertical Asymptote:

Horizontal Asymptote:

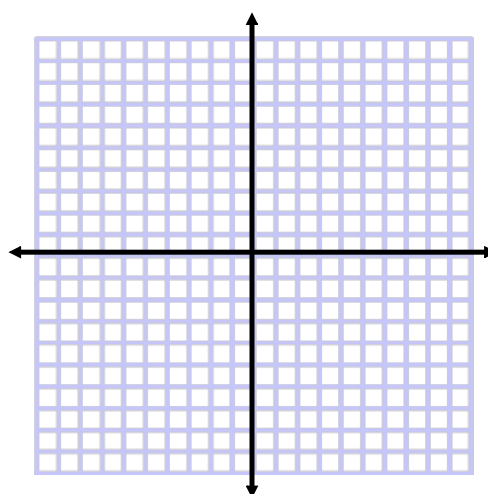
Example 1:

Graph the Reciprocal of a Linear Function

Consider $f(x) = 2x + 5$.

- Determine its reciprocal function .
- Determine the equation of the vertical asymptote of the reciprocal function.
- Graph the function $y = f(x)$ and its reciprocal function .

Describe a strategy that could be used to sketch the graph of a reciprocal function.



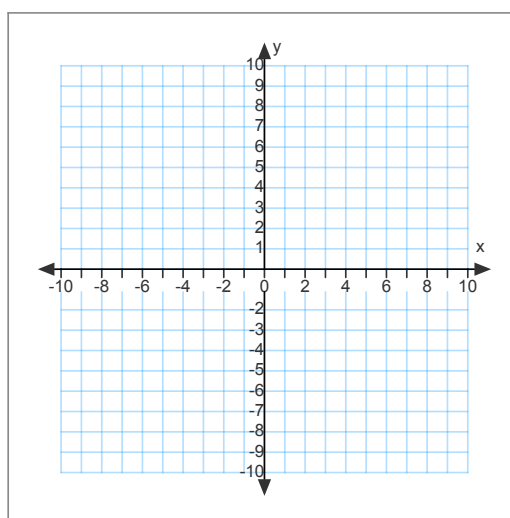
Example 2:

Consider $f(x) = 3x - 9$.

a) Determine its reciprocal function $y = \frac{1}{f(x)}$.

b) Determine the equation of the vertical asymptote of the reciprocal function.

c) Graph the function $y = f(x)$ and its reciprocal function $y = \frac{1}{f(x)}$, with and without technology. Discuss the behaviour of $y = \frac{1}{f(x)}$ as it nears its asymptotes.



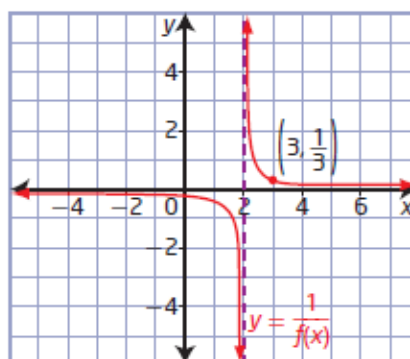
Example 3:

Graph $y = f(x)$ Given the Graph of $y = \frac{1}{f(x)}$

The graph of a reciprocal function of the form $y = \frac{1}{ax + b}$, where a and b are non-zero constants, is shown.

a) Sketch the graph of the original function, $y = f(x)$.

b) Determine the original function, $y = f(x)$.



Turn Over

Example 4:

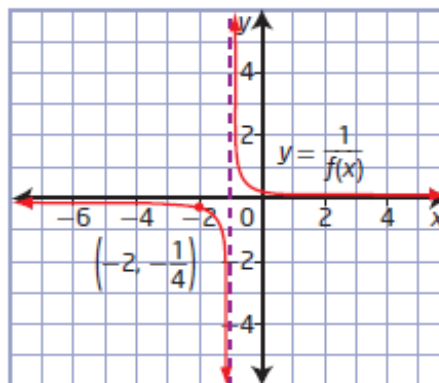
The graph of a reciprocal function of the form

$$y = \frac{1}{f(x)} = \frac{1}{ax + b}$$

where a and b are non-zero constants, is shown.

a) Sketch the graph of the original function, $y = f(x)$.

b) Determine the original function $y = f(x)$.



Assign p. 403, #'s 1(ab), 2(ab), 3(ab), 4, 6(a), 7(ac), 22

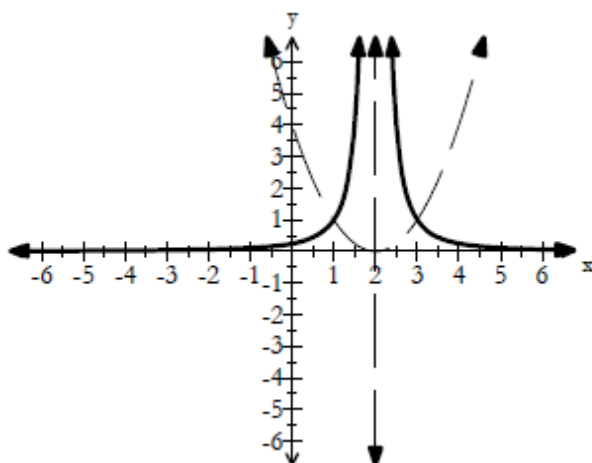
Part B: Reciprocal Functions of Quadratic Functions

There are 3 cases to consider, quadratic functions that have:

1. one x -intercept
2. two x -intercepts
3. no x -intercepts

CASE 1: Quadratic Functions with **one x -intercept**.

Example: Consider the graph of $y = (x - 2)^2$ and $y = \frac{1}{(x - 2)^2}$.



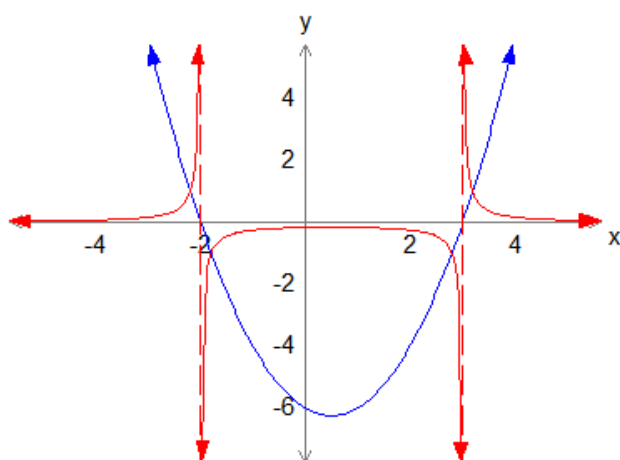
vertical asymptote

invariant points

horizontal asymptote

CASE 2: Quadratic Functions with **two x-intercepts**.

Example: Consider the graph of $y = (x-3)(x+2)$ and $y = \frac{1}{(x-3)(x+2)}$.



vertical asymptote

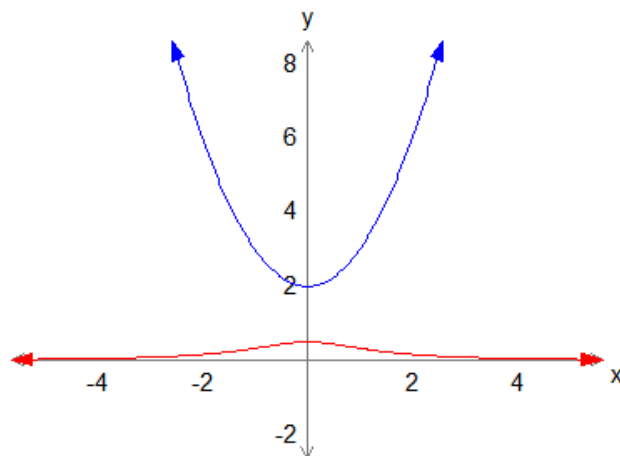
invariant points

horizontal asymptote

turn over

CASE 3: Quadratic Functions with NO x -intercepts.

Example: Consider the graph of $y = x^2 + 2$ and $y = \frac{1}{x^2 + 2}$.



vertical asymptote

invariant points

horizontal asymptote

KEY CONCEPTS:

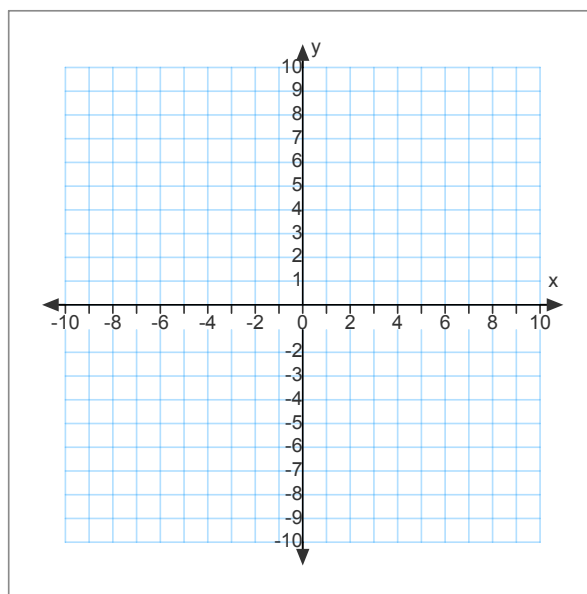
- Vertical asymptotes of the reciprocal function = the x -intercepts of the original function
- The point (x, y) on the reciprocal function becomes $(x, 1/y)$ on the original function.
- The two functions will intersect when $f(x) = \pm 1$.
- Horizontal asymptote occurs at $y = 0$.

Example 1:

Graph the Reciprocal of a Quadratic Function

Consider $f(x) = x^2 - 4$.

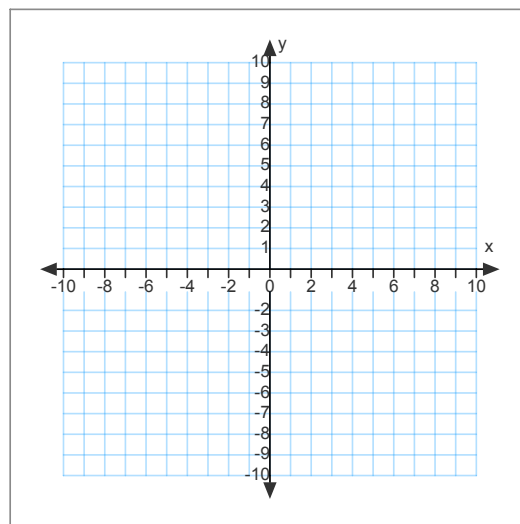
- What is the reciprocal function of $f(x)$?
- State the non-permissible values of x and the equation(s) of the vertical asymptote(s) of the reciprocal function.
- What are the x -intercepts and the y -intercept of the reciprocal function?
- What are the invariant points?
- Graph the function $y = f(x)$ and its reciprocal function $y = \frac{1}{f(x)}$.



Example 2:

Consider $f(x) = x^2 + x - 6$.

- What is the reciprocal function of $f(x)$?
- State the non-permissible values of x and the equation(s) of the vertical asymptote(s) of the reciprocal function.
- What are the x -intercepts and the y -intercept of the reciprocal function?
- Sketch the graphs of $y = f(x)$ and its reciprocal function $y = \frac{1}{f(x)}$



Key Ideas p. 402

Assign p. 403, #'s 1(cd), 2(cd), 3(cd), 6(bc), 8(ac), 9