Unit 5 - Radical Expressions and Equations

Recall from Grade 10:

$$\sqrt[index]{radicand} = radical$$

For instance, in the radical $\sqrt[3]{27}$

3 is the ***index*** and 27 is the ***radicand***

The first 10 perfect squares:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

The first 10 perfect cubes:

1,8,27,64,125,216,343,512,729,1000

Mixed and Entire Radicals

Examples:

1. Simplify in exact form :

$a)\ \sqrt{200}$ $b)\ \sqrt[3]{320}$ $c)\ -2\sqrt{27}$

$d)\sqrt[3]{-27}$ $e)\sqrt{-16}$

Conclusion:

- If a radical has an even index, then the radicand must be non-negative (positive)
- If a radical has an odd index, then the radicand can be any real number, including negative numbers

2. Express each mixed radical as an entire radical.

$a)\ 3\sqrt{5}$ $b)\ 5\sqrt[3]{4}$

Review!
Solve the following!

$$x^2 - 49 = 0$$

An important distinction! $7^2 = 49$ and $(-7)^2 = 49$

The positive square root is called the ***Principal Square Root***

$$\sqrt{49} = 7$$

The negative square root is called the ***Secondary Square Root***

$$-\sqrt{49} = -7$$

In certain situations (finding lengths) it makes sense to only take the principal square root.

Assign p. 278 #1, 2(abc)

Radicals Involving Variables

When a radical contains a variable, we must consider RESTRICTIONS on the variable.

(Are there any values of ***x*** that is not allowed?)

NOTE:
1. You cannot take the square root of a negative number!

2. You cannot divide by zero.

Let's take a look at the following:

$1.a)\ \sqrt{x}$ $\quad b)\ \sqrt{x-3}$ $\quad c)\ \sqrt{3x+2}$

What is the difference between (abc) and (def)?

How does that change its restrictions?

$d)\ \frac{1}{\sqrt{x}}$ $\quad e)\ \frac{1}{\sqrt{x-3}}$ $\quad f)\ \frac{1}{\sqrt{3x+2}}$

2. State the restrictions:

$$\frac{4}{\sqrt{2x-7}}$$

NOTE: Recall the fractional exponent from last year!

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$$

To convert a fractional exponent to a radical:

- the numerator of the fraction becomes the exponent of the radicand/radical
- the denominator of the fraction becomes the index of the radical

Restrictions ➡ Odd/even

What are the restrictions of each of the following?

1) $\sqrt{x^4}$ 2) $\sqrt{x^6}$ 3) $\sqrt{x^7}$

Examples: Simplify. What are the restrictions on each variable?

1. $a)\ \sqrt{3x^4y^5}$ $b)\ \sqrt[5]{a^2b^3}$

$c)\ \sqrt{x^{13}}$ $d)\ \sqrt{12x^{25}y^{16}}$

Now we can extend this to examples that include mixed radicals:

2.*a*) $2\sqrt{28x^{13}}$ *b*) $3x^2y^3\sqrt{18x^8y^2}$

c) $\sqrt[4]{x^9}$

What are the restrictions on each variable above?

3. Express the mixed radical to an entire radical.

a)$a^4\sqrt{a^5}$ *b*)$5b^3\sqrt[3]{3b^2}$

Summary of restrictions of radicals!

- If a radical has an even index and
- radicand has an even exponent then $x \in \Re$
- radicand has an odd exponent then $x \geq 0$
- If a radical has an odd index then $x \in \Re$

Key Ideas p. 278

Assign: p. 278-281 #2d,3,4 worksheet