


3.3 Completing the Square

In Section 3.2, we simplified the vertex form and obtained standard form:

For example,	$y = 2(x - 3)^2 + 4$	Vertex Form	
	$y = 2(x - 3)(x - 3) + 4$	Expand	
	$y = 2(x^2 - 6x + 9) + 4$	FOIL	
	$y = 2x^2 - 12x + 18 + 4$	Simplify	
	$y = 2x^2 - 12x + 22$	Standard Form	

$$y = ax^2 + bx + c$$

Now, we will look at going from:

standard form \longrightarrow vertex form

by using a process called *completing the square*.

The advantage of writing a quadratic in vertex form is to easily identify the vertex to solve max/min word problems.

To complete the perfect square trinomial, add the square of half the coefficient of the x-term. In other words.

$$c = \left(\frac{b}{2}\right)^2 = \left(\frac{1}{2}b\right)^2$$

Determine the value that should be placed in each blank in order to create a perfect square trinomial:

a) $x^2 + 4x + \underline{\quad}$ b) $x^2 - 8x + \underline{\quad}$ c) $x^2 - 14x + \underline{\quad}$

d) $x^2 - 2x + \underline{\quad}$ e) $x^2 - 3x + \underline{\quad}$ f) $x^2 + 9x + \underline{\quad}$

3.3

Example 1

Convert From Standard Form to Vertex Form

Rewrite each function in vertex form by completing the square and state the vertex.

a) $f(x) = x^2 - 8x + 5$ NOTE: $a = 1$

b) $f(x) = 3x^2 - 12x - 9, a \neq 1$

Turn over

3.3 Example 1 Continued

c) $f(x) = -5x^2 - 70x$

Example 2

Convert the function $y = 4x^2 - 28x - 23$ to vertex form.

Assign p. 192 – 197, #2(ab), 3(bd), 4(ab), 6(bd), 7(bc), 12(cd)

Domino Activity

3.2

MAX/MIN Word Problems

Example 1 (p. 168)

Analysing a Quadratic Function

A frog sitting on a rock jumps into a pond. The height, h , in centimetres, of the frog above the surface of the water as a function of time, t , in seconds, since it jumped can be modelled by the function $h(t) = -490t^2 + 150t + 25$. Where appropriate, answer the following questions to the nearest tenth.

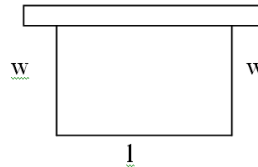
- What is the y -intercept? What does it represent in this situation?
- What maximum height does the frog reach? When does it reach that height?
- How high is the frog 0.25 s after it jumps?



Turn over

Example 2:

A gardener uses 60 m of bordering to form a rectangular garden. One side of the garden touches a wall and doesn't need any bordering. Algebraically determine the quadratic function that models the area of the garden and use it to determine the dimensions of the garden which result in it having the greatest area?



Example 3:

Determine two numbers that differ by 28 and whose product is a minimum.

Turn over

Example 4:

The student council at a high school is planning a fundraising event with a professional photographer taking portraits of individuals or groups. The student council gets to charge and keep a session fee for each individual or group photo session. Last year, they charged a \$10 session fee and 400 sessions were booked. In considering what price they should charge this year, student council members estimate that for every \$1 increase in the price, they expect to have 20 fewer sessions booked.

- a) Write a function to model this situation.
- b) What is the maximum revenue they can expect based on these estimates. What session fee will give that maximum?

Turn over

Example 3 (p. 171)

Write a Quadratic Function to Model a Situation

A rancher has 100 m of fencing available to build a rectangular corral.

- a) Write a quadratic function in standard form to represent the area of the corral.
- b) What are the coordinates of the vertex?
What does the vertex represent in this situation?
- c) Sketch the graph for the function you determined in part a).
- d) Determine the domain and range for this situation.
- e) Identify any assumptions you made in modelling this situation mathematically.



Turn over

3.2

Example 3: Your Turn

At a children's music festival, the organizers are roping off a rectangular area for stroller parking. There is 160 m of rope available to create the perimeter.

- a) Write a quadratic function in standard form to represent the area for the stroller parking.
- b) What are the coordinates of the vertex? What does the vertex represent in this situation?
- c) Sketch the graph for the function you determined in part a).
- d) Determine the domain and range for this situation.
- e) Identify any assumptions you made.

Key Ideas p. 192

Assign p. 174 – 176, #7, 12bcf, 17

p. 194 - 197, #13, 14, 18, 19, 21

Max/Min Worksheet