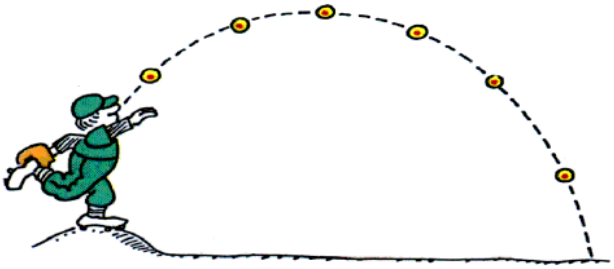
# Unit 3 - Quadratic Functions

## 3.1 Investigating Quadratic Functions in Vertex Form

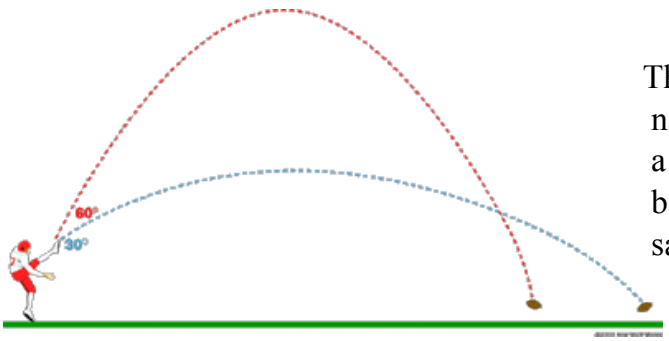
Math 1201: Linear Functions

Math 2200: Quadratic Functions

Math 3200: Cubic, Quartic, Quintic Functions



The path a ball travels gives a special "U" shape called a "***parabola***."




This parabolic shape occurs in many natural phenomena such as kicking a football/soccer ball, hitting a golf ball, flight path of birds, parabolic satellites, etc.

## *Quadratic Functions:*

⟶ the shape is a parabola

⟶ the simplest quadratic function is $y = x^2$

(The word *quadratic* comes from the word *quadratum*, a Latin word meaning *square*.)

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How to create a quadratic function?

What do you notice about the degree (highest exponent of the variable) of the function?

Which of the following functions are quadratic?

i) $y = 5(x+3)$

ii) $y = 5x(x+3)$

iii) $y = 5(x^2+3)$

iv) $y = (5x+1)(x+3)$

v) $y = 5^2(x+3)$

vi) $y = 5(x+3)^2 + 2$

⟶

vii) $y = 4(x+3)(x-1)(x+2)$

vii) $y = 3(2-x)(1-x)$

Characteristics of the basic quadratic function $y = x^2$.

Create table of values

| | |
|---|---|
| | |
| | |
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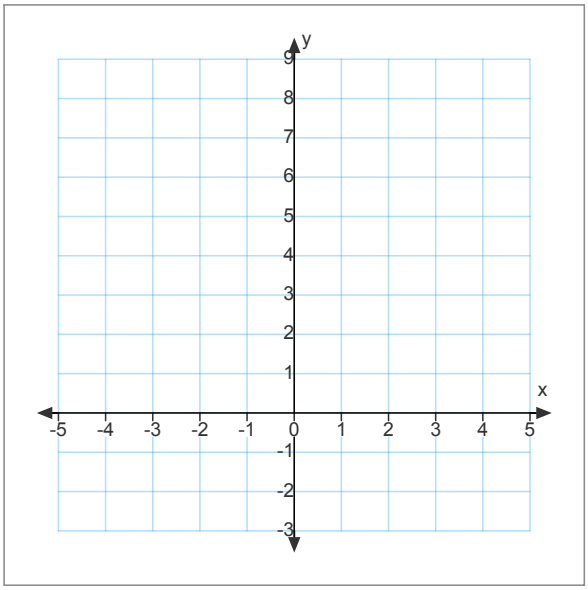
| | |




What is the vertex?

What is the x-intercept?

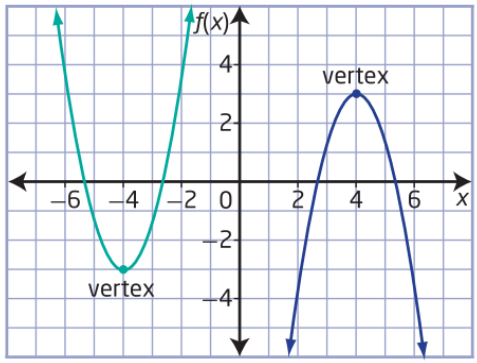
What is the y-intercept?

What is the domain and range?

Can you visualize other types of parabolas?

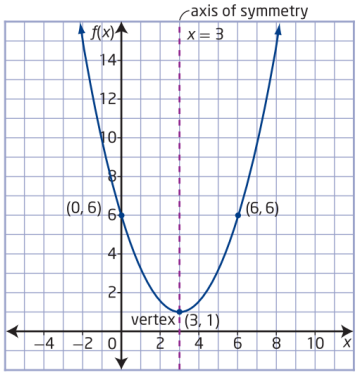
***Direction of Opening: parabola that can open up or down.***

- When the graph opens **up** the vertex is the **lowest** point on the graph and the $y$-coordinate of the vertex is the **minimum** value
- When the graph opens **down**, the vertex is the **highest** point on the graph and the $y$-coordinate of the vertex is the **maximum** value.
- The vertex is labelled as $(p, q)$




***Axis of Symmetry***

- The parabola is symmetric about a line called the **axis of symmetry**
- This lines divides the graph into two equal parts
- It is a mirror image
- It intersects the parabola at the **vertex**
- The equation of the axis of symmetry corresponds to the $x$-coordinate of the vertex $x = p$



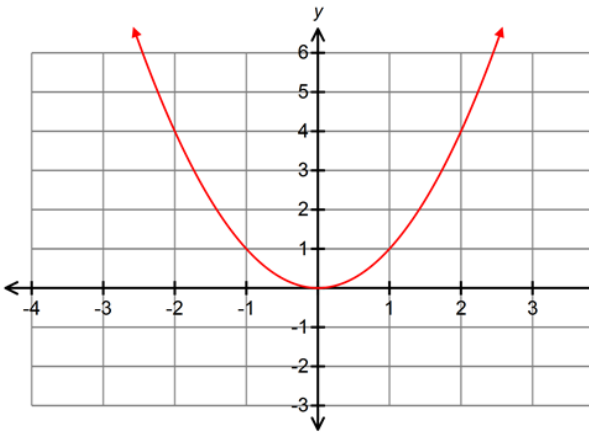
### *Relation vs Function*

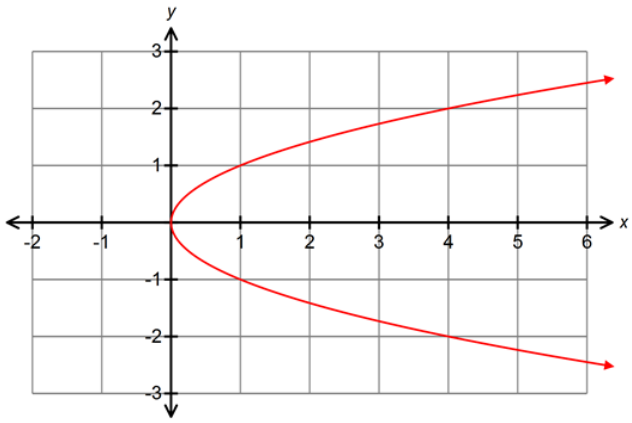
Why are quadratic *relations* also quadratic *functions*?

> For every value of $x$ there is only one value for $y$.

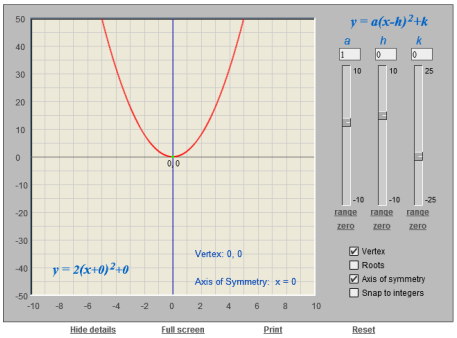
> It passes the vertical line test!

Think about:



# Investigating the Vertex Form of Quadratic Functions




http://www.mathopenref.com/quadvertexexplorer.html

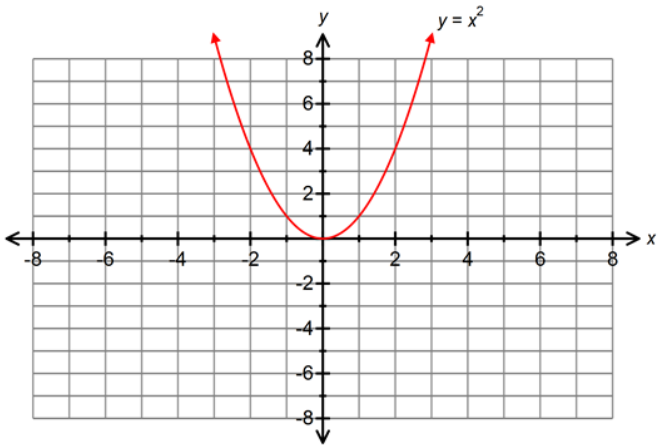
***Quadratic Function:*** $y = a(x-p)^2 + q$

I. The Effect of Parameter ***a*** in $y = ax^2$ on the graph of $y = x^2$

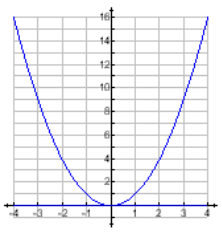
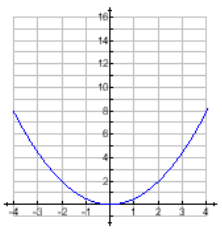
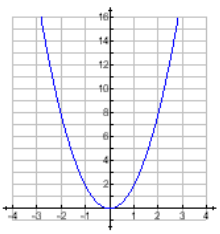
$y = x^2$

$y = 0.5x^2$

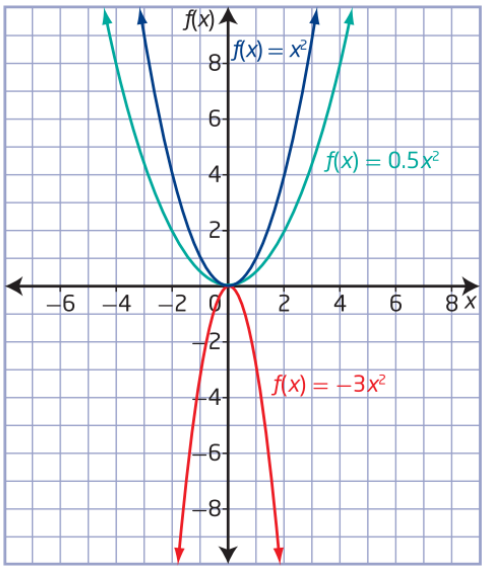
$y = -3x^2$




Notice how the co-ordinates change.

| $y = x^2$ | $y = \frac{1}{2}x^2$ | $y = 2x^2$ |
|---|---|---|
|  |  |  |

- ***a*** determines the:
  - > shape of the parabola
  - > direction of opening
  - > vertical stretch
  - > reflection on $x$-axis




Summary:

| If $a > 0$ ($a$ is positive) | If $a < 0$ ($a$ is negative) | If $-1 < a < 1$ | If $a > 1$ or $a < -1$ |
|---|---|---|---|
| it opens up and has a ***minimum*** point | it opens ***down*** and has a ***maximum*** point; reflection on $x$-axis | the parabola is ***wider*** compared to the graph of $y = x^2$ | the parabola is ***narrower*** compared to the graph of $y = x^2$ |

Example: Compare $y = -\frac{1}{2}x^2$ to the function $y = x^2$

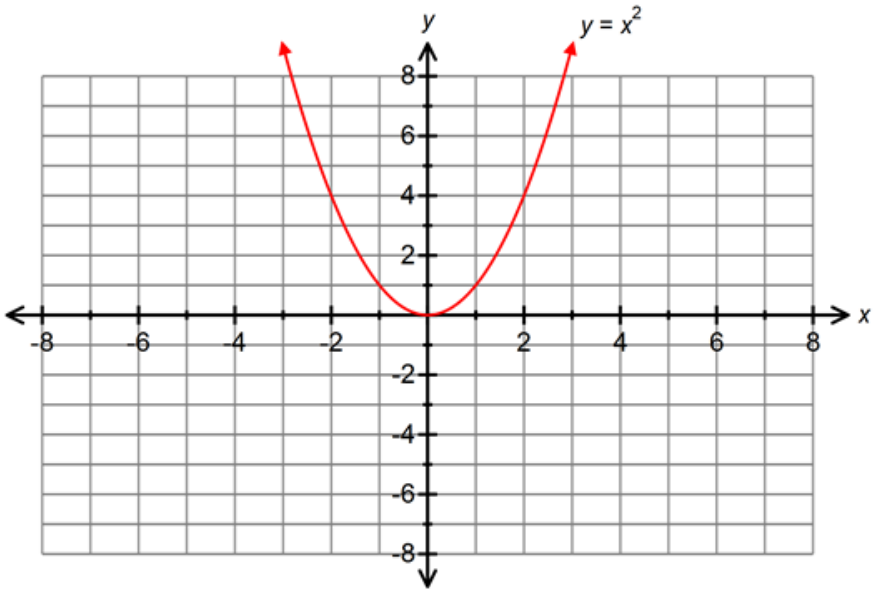
What is the vertical stretch?

B. The effect of parameter ***q*** in $y = x^2 + q$ on the graph of $y = x^2$

$y = x^2$

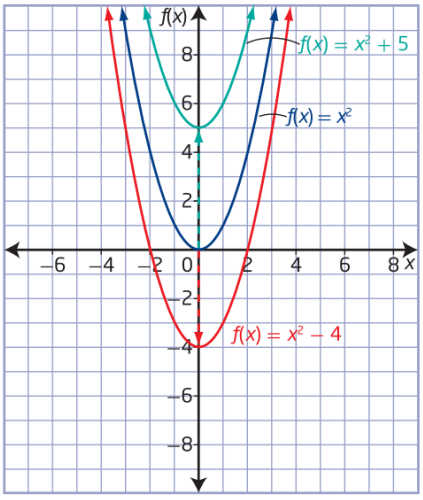
$y = x^2 + 5$

$y = x^2 - 4$




***What is the effect of q*** ?

- translates the parabola ***vertically (up/down)***
- $q$ = $y$-coordinate of vertex

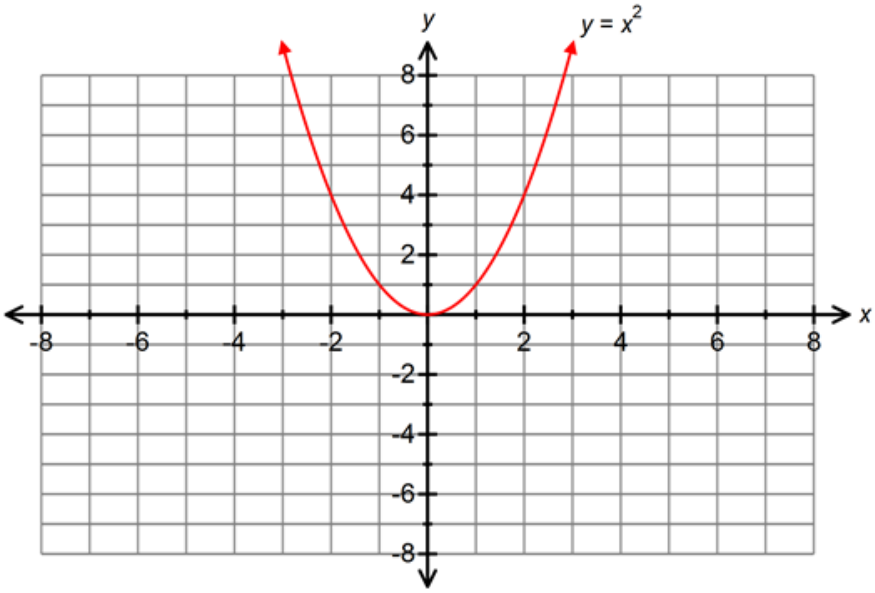


C. The Effect of Parameter ***p*** in $y=(x-p)^2$ on the graph of $y=x^2$

$y=x^2$

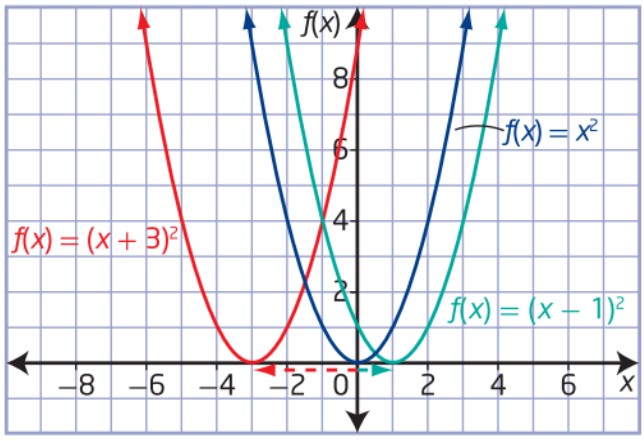
$y=(x+3)^2$

$y=(x-1)^2$

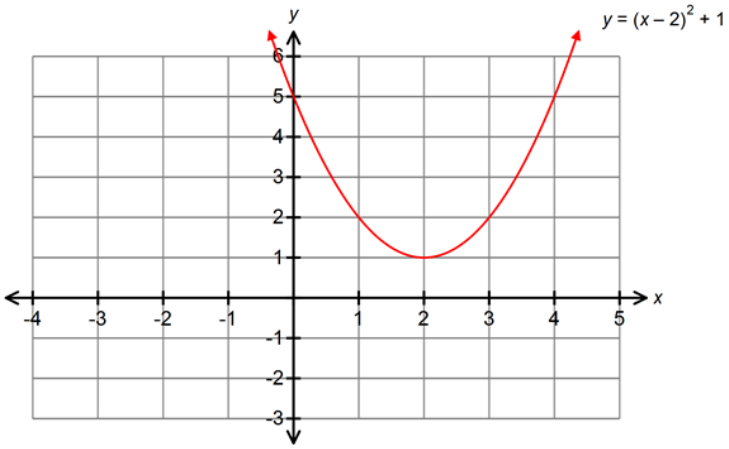



***What is the effect of p?***

- translates the parabola ***horizontally (left/right)***
- $p$ = $x$-coordinate of vertex
- the equation of the axis of symmetry is $x = p$



Example: What is the vertex of $y = (x-2)^2 + 1$ ?




***Domain and Range:***

- Domain for all quadratic functions is $\{x \in \Re\}$
- Range depends on two things:
  - vertex
  - direction of opening
- if $a > 0$, then $y \geq q$
- if $a < 0$, then $y \leq q$

Example:

What is the domain and range of the quadratic function if the vertex is (-3, 5) and opens downward?

***Assign p. 157, #1, 2, 12, 19***

State the following characteristics for each quadratic function.

| vertex | axis of symmetry | direction of opening narrower/wider |
|---|---|---|
| | equation $y = 2(x-3)^2 + 1$ | |
| maximum/minimum value | | number of x-intercepts |
| | domain: | |
| | range: | |

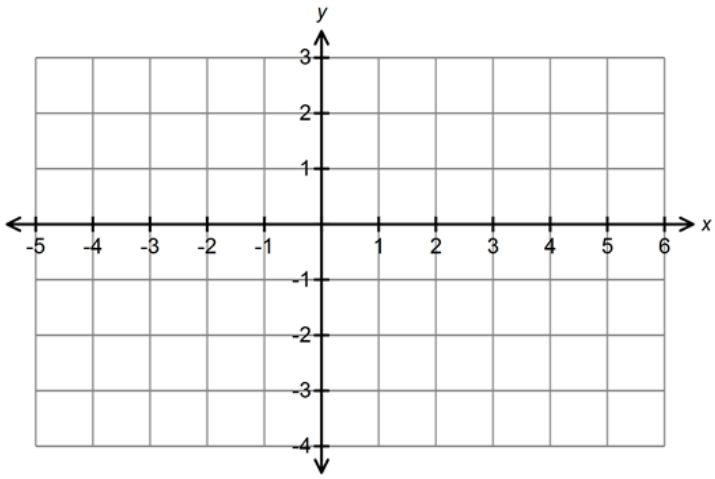
| vertex | axis of symmetry | direction of opening narrower/wider |
|---|---|---|
| | equation $y = -\frac{1}{2}(x+1)^2 - 3$ | |
| maximum/minimum value | | number of x-intercepts |
| | domain: | |
| | range: | |

**Example 1: Sketch Graphs of Quadratic Functions in Vertex Form**

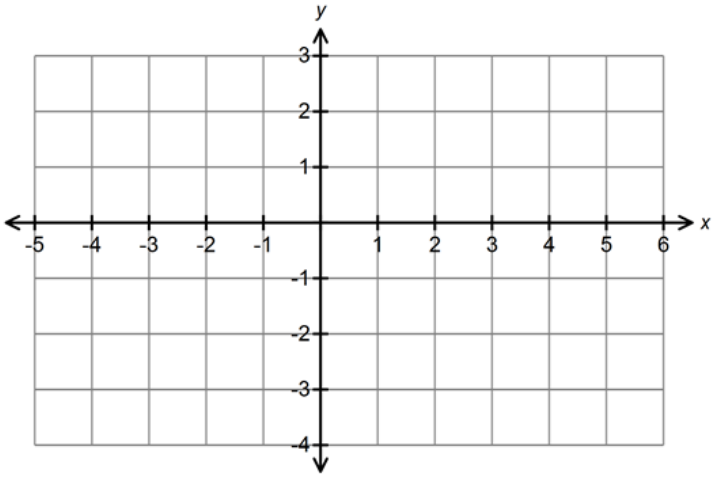
Determine the following characteristics for each function.
- the vertex
- the domain and range
- the direction of opening
- the equation of the axis of symmetry and sketch each graph.

$y = 2(x + 1)^2 - 3$

Method 1: using transformations



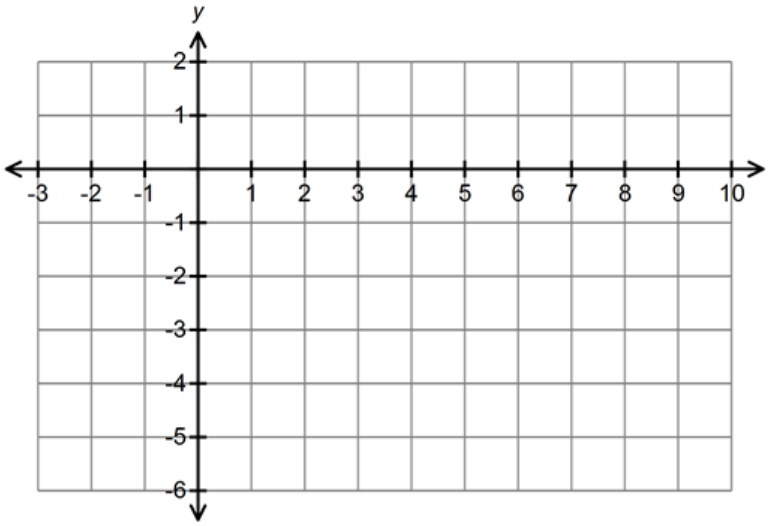

Method 2: using points and symmetry



## Example 2:

Determine the following characteristics for each function.
- the vertex
- the domain and range
- the direction of opening
- the equation of the axis of symmetry and sketch each graph.

$y = -\frac{1}{4}(x-4)^2 + 1$



## Your Turn

Determine the following characteristics for each function.
- the vertex
- the domain and range
- the direction of opening
- the equations of the axis of symmetry

Then, sketch each graph.

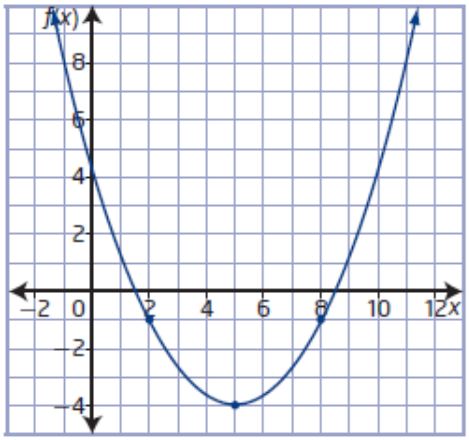
**a)** $y = \frac{1}{2}(x - 2)^2 - 4$

**b)** $y = -3(x + 1)^2 + 3$

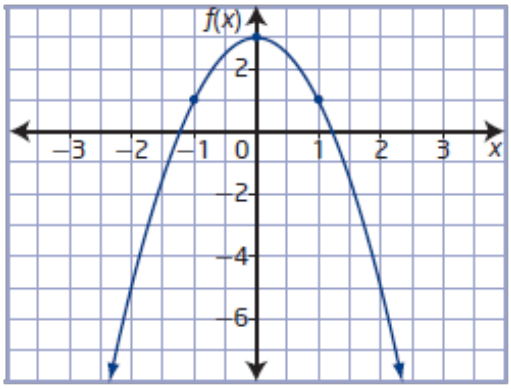
***Assign p. 157, #4(abc),10***

## Determine a Quadratic Function in Vertex Form Given Its Graph

**Example 1:**



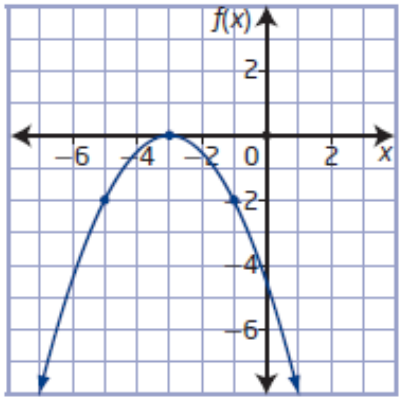

**Example 2:**



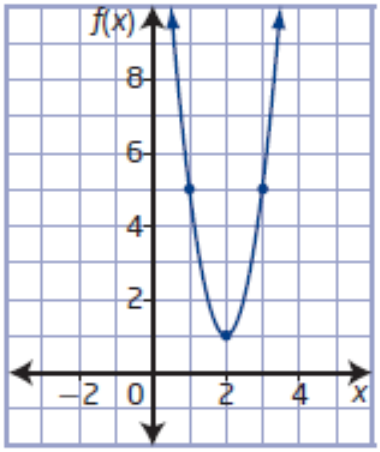
## Example 3: Your Turn

Determine a quadratic function in vertex form for each graph.

a)




b)




### *Example 4:*

Determine a quadratic function in vertex form given the maximum value is 5 and $x$-intercepts are -3 and 7.

***Assign:***

***p. 157, #8ac, 9, 21***

**Determine the Number of *x*-Intercepts without graphing the quadratic function.**

Identify the value of $a$ ⟶

Identify the value of $q$ ⟶

Example:
Determine the number of *x*-intercepts for each quadratic function.

**a)** $f(x) = 0.8x^2 - 3$

**b)** $f(x) = 2(x - 1)^2$

**c)** $f(x) = -3(x + 2)^2 - 1$

⟶

## Number of $x$-intercepts

| Value of $a$ | Value of $q$ | Number of $x$-intercepts |
|---|---|---|
| Positive | Positive | |
| Positive | Negative | |
| Positive | 0 | |
| Negative | 0 | |
| Negative | Positive | |
| Negative | Negative | |

## Example: Your Turn

Determine the number of $x$-intercepts for each quadratic function without graphing.

**a)** $f(x) = 0.5x^2 - 7$

**b)** $f(x) = -2(x + 1)2$

**c)** $f(x) = -\frac{1}{6}(x - 5)^2 - 11$

***Key Ideas p. 156***

***Assign p. 157, #6,7(abc)***

**Frayer Model Puzzle Activity**

**Guess the Graph**