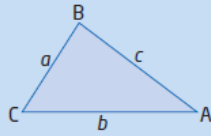


## 2.4 The Cosine Law

The COSINE law is another equation that can be used to find missing side lengths and angle measures.

### The Cosine Law

The **cosine law** describes the relationship between the cosine of an angle and the lengths of the three sides of any triangle.



For any  $\triangle ABC$ , where  $a$ ,  $b$ , and  $c$  are the lengths of the sides opposite to  $\angle A$ ,  $\angle B$ , and  $\angle C$ , respectively, the cosine law states that

$$c^2 = a^2 + b^2 - 2ab \cos C$$

You can express the formula in different forms to find the lengths of the other sides of the triangle.

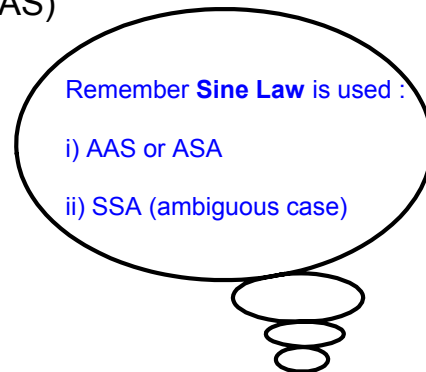
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

What patterns do you notice?

The Cosine Law is used when the following information is known:

- i) two sides and the included angle (SAS)
- ii) all three sides (SSS)



### Example 1

#### Determine a Distance

A surveyor needs to find the length of a swampy area near Fishing Lake, Manitoba. The surveyor sets up her transit at a point A. She measures the distance to one end of the swamp as 468.2 m, the distance to the opposite end of the swamp as 692.6 m, and the angle of sight between the two as  $78.6^\circ$ . Determine the length of the swampy area, to the nearest tenth of a metre.

\*

Example 2

Nina wants to find the distance between two points, A and B, on opposite sides of a pond. She locates a point C that is 35.5 m from A and 48.8 m from B. If the angle at C is  $54^\circ$ , determine the distance AB, to the nearest tenth of a metre.

Example 3

**Determine an Angle**

The Lions' Gate Bridge has been a Vancouver landmark since it opened in 1938. It is the longest suspension bridge in Western Canada. The bridge is strengthened by triangular braces. Suppose one brace has side lengths 14 m, 19 m, and 12.2 m. Determine the measure of the angle opposite the 14-m side, to the nearest degree.

\*

Example 4

**Solve a Triangle**

In  $\triangle ABC$ ,  $a = 11$ ,  $b = 5$ , and  $\angle C = 20^\circ$ . Sketch a diagram and determine the length of the unknown side and the measures of the unknown angles, to the nearest tenth.

Key Ideas Summary p. 119

Assign p. 119-125 questions 1(ac), 2(a), 3, 4(ce), 7, 9, 12, 30