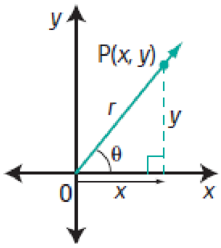
2.2 Trigonometric Ratios of Any Angle

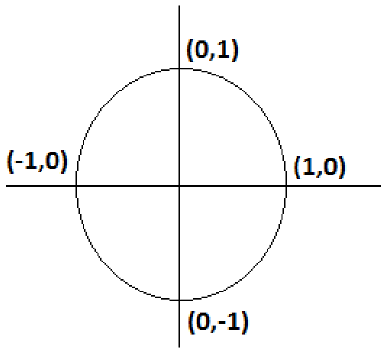
Finding the Trigonometric Ratios of Any Angle θ, where $0° \leq \theta < 360°$



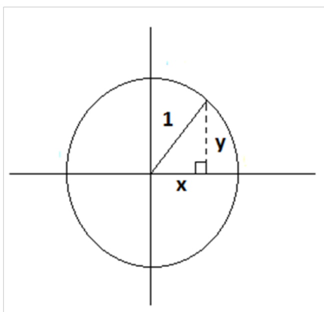
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

Unit Circle - a circle with a radius of 1



If we use a unit circle with r = 1, we get:



Note that the point on the unit circle (x,y) is now expressed as (cos, sin)

Therefore, the ratios are:

$$cos\theta = \frac{x}{r} = \frac{x}{1} = x$$

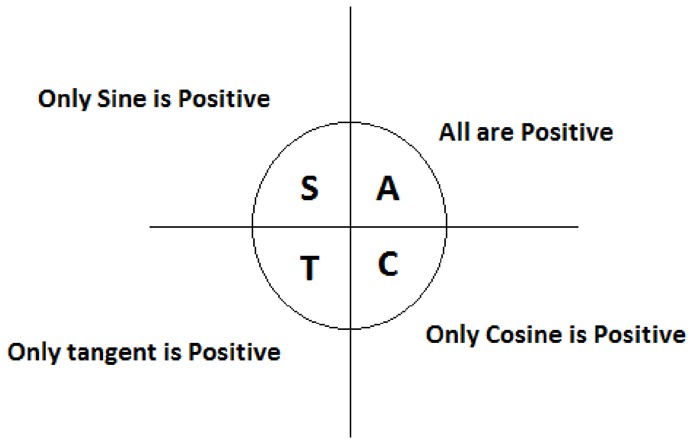
$$sin\theta = \frac{y}{r} = \frac{y}{1} = y$$

Complete the chart to show the signs of the trigonometric ratios in each quadrant.

Quadrant	Sin	Cos	Tan
1			
2			
3			
4			

CAST Rule

The CAST Rule identifies the quadrants where the primary trigonometric ratios are **positive**.



EX.

In which quadrant is sine positive and cosine negative?

In which quadrant is tangent positive and cosine negative?

Signs of the Trigonometric Ratios

Check or circle the ratios that are positive within each of the quadrants.

II	I
sin θ cos θ tan θ	sin θ cos θ tan θ
sin θ cos θ tan θ	sin θ cos θ tan θ
III	IV

Example 1

Write Trigonometric Ratios for Angles in Any Quadrant

The point P(–8, 15) lies on the terminal arm of an angle, θ, in standard position. Determine the exact trigonometric ratios for sin θ, cos θ, and tan θ.

Example 3

Determine Trigonometric Ratios

Suppose θ is an angle in standard position with terminal arm in quadrant III, and $\cos\theta = -\frac{3}{4}$. What are the exact values of sin θ and tan θ?

*

When two radicals are divided and the numbers under the radical signs cannot be divided evenly, we must use a process called...

Rationalizing the Denominator

...which converts the denominator to a rational number.

** Multiply the numerator and the denominator by the radical in the denominator.

$$\frac{2}{\sqrt{5}} \qquad \frac{\sqrt{7}}{\sqrt{3}}$$

When two mixed radicals are divided, we can divide the "outsides" and divide the radicals.

$$\frac{24\sqrt{14}}{3\sqrt{2}} \qquad \frac{6\sqrt{30}}{-12\sqrt{3}}$$

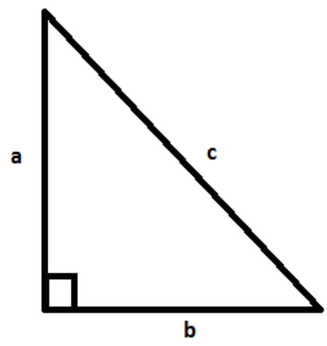
Mixed radicals may also require Rationalizing of the Denominator if the radicals do not divide evenly.

$$\frac{7\sqrt{10}}{5\sqrt{3}}$$

Multiply by

$\sqrt{3}$ not $5\sqrt{3}$

In a right-angled triangle, we use the abbreviation **SOHCAHTOA** to help us define the primarytrigonometric ratios.



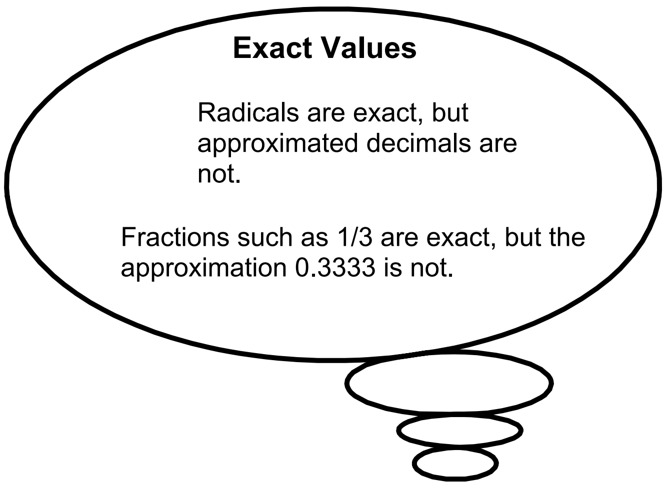
$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

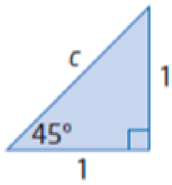
$$tan\theta = \frac{opposite}{adjacent}$$

Special Right Triangles

For angles of 30^0, 45^0, and 60^0 you can determine the **exact values** of trigonometric raos.



$45^o - 45^o - 90^o$ Triangle



$\sin 45^o =$ $\cos 45^o =$ $\tan 45^o =$

$30^o - 60^o - 90^o$ Triangle

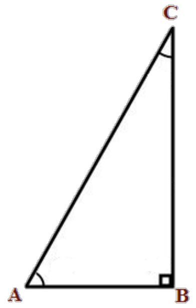
Draw an equilateral Triangle with side length 2.

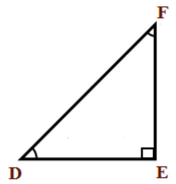
$\sin 60^o =$ $\cos 60^o =$ $\tan 60^o =$

$\sin 30^o =$ $\cos 30^o =$ $\tan 30^o =$

Special Right Triangles

For angles of 30°, 45°, and 60°, you can determine the exact values of trigonome ratios. Determine what should go in each cell, and then click on the button in each cell to check.





θ	sin θ	cos θ	tan θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°		$\frac{1}{\sqrt{2}}$	
60°			

Example 2

Determine the Exact Value of a Trigonometric Ratio

Determine the exact value of cos 135°.

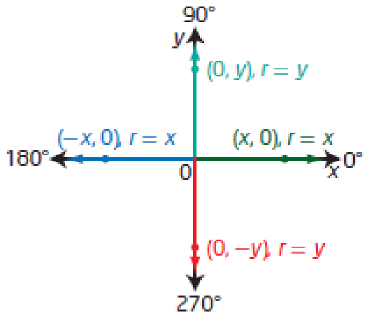
Example 4

Determine Trigonometric Ratios of Quadrantal Angles

Determine the values of sin θ, cos θ, and tan θ when the terminal arm of quadrantal angle θ coincides with the positive *y*-axis, θ = 90°.

*

Use the diagram to determine the values of sin θ, cos θ, and tan θ for quadrantal angles of 0°, 180°, and 270°. Organize your answers in a table as shown below.



	0°	90°	180°	270°
sin θ		1		
cos θ		0		
tan θ		undefined		

Example 5

Solve for an angle Given Its Exact Sine, Cosine or Tangent Value

Solve for θ.

a) $\sin\theta = 0.5,\ 0^\circ \le \theta < 360^\circ$

b) $\cos\theta = -\frac{\sqrt{3}}{2}, 0^\circ \le \theta \le 360^\circ$

*

Example

Point (2,4) lies on the terminal arm of angle θ, in standard position.

a) What is the measure of the reference angle, to the nearest tenth?

b)What is the measure of θ, to the nearest degree?

Example

Given $\cos \theta = -0.6753$, where $0^\circ \le \theta < 360^\circ$, determine the measure of θ, to the nearest tenth of a degree.

*

Example 7

Find an Exact Distance

Allie is learning to play the piano. Her teacher uses a metronome to help her keep time. The pendulum arm of the metronome is 10 cm long. For one particular tempo, the setting results in the arm moving back and forth from a start position of 60° to 120°. What horizontal distance does the tip of the arm move in one beat? Give an exact answer.

Example 8

The arm of a crane used for lifting very heavy objects can move so that it has a minimum angle of inclination of 30° and a maximum of 60°. Use exact values to find an expression for the change in the vertical displacement of the end of the arm, in terms of the length of the arm, *a*.

Key Ideas Summary p. 95

Assign
p. 96-99
questions 2(bc),3(bc),5(ab),7,8(abc),9(abdf),10,12,15,16,24,29

p. 84
questions 11,13

Snap Activity