

## 1.2 Arithmetic Series

### *Arithmetic Series:*

- a **sum** of terms in an arithmetic sequence.

Example: arithmetic sequence  $\{3, 5, 7, 9, 11, 13\}$

arithmetic series  $3 + 5 + 7 + 9 + 11 + 13$

- $S_n$  represents the sum of the first  $n$  terms and is read "S sub n"

Example: sum of the first three terms:  $S_3 =$

sum of the first 4 terms:  $S_4 =$

*Derive a formula to determine the sum of a finite arithmetic series.*

**Example 1:** What is the sum of the first 5 natural numbers?

**Example 2:** What is the sum of the first 100 natural numbers?

$S_{100} = 1 + 2 + 3 + \dots + 98 + 99 + 100$

Generalize:

Formula:

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**Alternate Form:**

$$S_n = \frac{n}{2}(t_1 + t_n) \quad \text{or} \quad S_n = \frac{n(t_1 + t_n)}{2}$$

Let's look at another way to represent the formula for the sum of an arithmetic series. Determine the expression that can be substituted in for  $t_n$ ?

**Proof:**

$$\begin{aligned} S_n &= t_1 + (t_1 + d) + \cdots + [t_1 + (n-2)d] + [t_1 + (n-1)d] \\ S_n &= [t_1 + (n-1)d] + [t_1 + (n-2)d] + \cdots + (t_1 + d) + t_1 \\ \hline 2S_n &= [2t_1 + (n-1)d] + [2t_1 + (n-1)d] + \cdots + [2t_1 + (n-1)d] + [2t_1 + (n-1)d] \\ 2S_n &= n[2t_1 + (n-1)d] \\ S_n &= \frac{n}{2}[2t_1 + (n-1)d] \end{aligned}$$

**Example 3:**

An auditorium has 20 seats on the first row, 24 seats on the second row, 28 seats on the third row, and so on, and has 30 rows of seats. How many seats are there in the theatre?

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Example 4:

Determine the sum of  $20 + 14 + 8 + \dots + (-70)$ .

Example 5:

In an arithmetic series  $t_1 = 6$  and  $S_9 = 108$ .

Determine the common difference and the sum of the first 20 terms.

Example 6: (#7a p. 27)

Determine the sum of all the multiples of 4 between 1 and 999.

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Example 7: (p.25 resource)



Male fireflies flash in various patterns to signal location or to ward off predators. Different species of fireflies have different flash characteristics, such as the intensity of the flash, the rate of the flash, and the shape of the flash. Suppose that under certain circumstances, a particular firefly flashes twice in the first minute, four times in the second minute, and six times in the third minute.

- a) If this pattern continues, what is the number of flashes in the 30th minute?
- b) What is the total number of flashes in 30 min?

Example 8: (like #10, p. 28)

If  $t_5 = 31$  and  $t_{16} = 97$ , determine the sum of the first 40 terms of the arithmetic sequence.

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Pre-assessment:

Solve the following: 
$$\left. \begin{array}{l} 2x + y = 1 \\ 3x - 2y = 19 \end{array} \right\}$$

Example 9:

The sum of the first two terms of an arithmetic series is 13 and the sum of the first four terms is 46. Determine the first six terms of the series and the sum to six terms.

Example 10:

The sum of the first two terms of an arithmetic series is 19 and the sum of the first four terms is 50. What are the first six terms of the series and the sum to 20 terms?

Key Ideas p. 27

Assign p. 27 - 30

1(ac), 2(bd), 3(bc), 4(ad), 5(a), 6(ac), 7(b), 10, 11, 14, 15, 22