

Sequences and Series

1.1 Arithmetic Sequences

Sequence:

- An ordered list of numbers in which each item or term follows another according to a pattern or rule to determine the next in the sequence.

Example 1: {2, 5, 8, 11 } What is the pattern?

Example 2: {-8, -6, -4, -2} What is the pattern?

Term (Element):

- Each item in the sequence.
- labelled according to their position.

t_1 :represents the first term

t_2 :represent the second term

t_n :represents the n^{th} term (read "t sub n")

Example 3: Label the terms in the above example: {2, 5, 8, 11, . . . }

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Finite vs. Infinite Sequences

1. Finite Sequence:

- has a finite number of terms
- it will eventually end (has a last term)

Ex: {2,5,8,11,14} and {5,10,15,20,...100}

2. Infinite Sequence

- has an infinite number of terms (indicated by ...)

Ex: {2,5,8,11,14,...}

Arithmetic Sequences:

- sequences that increase or decrease by a constant amount

Example: {5, 12, 19, 26}

- this constant amount is called a common difference (d)
- the difference between any two consecutive terms gives the common difference (subtract the first term from the second)

$$t_2 - t_1 =$$

$$t_3 - t_2 =$$

$$t_4 - t_3 =$$

Common Difference (d):

- can be positive or negative. Why?
- found by subtracting an element from the previous element.

Example 4: {8, 5, 2, -1 } What is the common difference?

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General Term:

- an expression to determine any term of a sequence.
- represented by t_n

Example 5: Consider the arithmetic sequence $\{6, 13, 20, 27, \dots\}$

Identify a pattern for the n^{th} term using the first term and the common difference:

$$t_1 =$$

$$t_2 =$$

$$t_3 =$$

$$t_4 =$$

$$t_5 =$$

.

.

$$t_n =$$

Formula for the General Term of an Arithmetic Sequence:

The *general term* of an arithmetic sequence is

$$t_n = t_1 + (n - 1)d$$

where t_1 is the first term of the sequence

n is the number of terms

d is the common difference

t_n is the general term or n^{th} term

Example 6: Given the sequence $\{27, 23, 19, 15, \dots\}$

a) Is the sequence an arithmetic sequence? Explain.

b) Determine the formula for the general term.

c) Use the relation rule to find t_{20} .

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Notation: Distinguish between t_{10} vs. $t_n = 10$

t_{10} :

Ex: $\{-8, -4, 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40\}$

$t_n = 10$

Ex: $\{4, 7, 10\}$

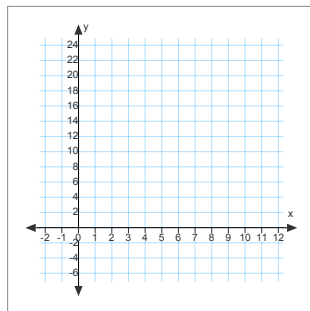
Graphing an Arithmetic Sequence:

Example 7:

(a) Write the general term of the sequence $\{-6, -2, 2, 6\dots\}$

(b) Determine the value of the 8th term.

(c) Graph the terms. What do you notice? Discrete or Continuous?



Domain:

Range:

Slope:

y-intercept:

(d) How is the domain of an arithmetic sequence different from the domain of a linear function?

(e) Describe the relationship between the slope of the graph and the formula from part (a)

(f) Describe the relationship between the y-intercept and the formula from (a)

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Example 8:

Is $\{2x + 5, 4x + 5, 6x + 5, \dots\}$ arithmetic? Explain.

Example 9:

Consecutive terms of an arithmetic sequence are $(5 + x)$, 8 , $(1 + 2x)$. What is the value of x ?

Example 10:

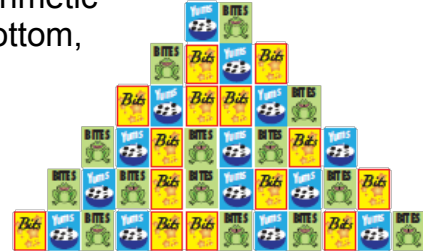
Consider the sequence $\{17, 21, 25, \dots, 157\}$. Determine the number of terms in the sequence.

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Example 11: (page 13 in resource)

Determine t_1 , t_n , and n

Jonathon has a part-time job at the local grocery store. He has been asked to create a display of cereal boxes. The top six rows of his display are shown. The numbers of boxes in the rows produce an arithmetic sequence. There are 16 boxes in the third row from the bottom, and 6 boxes in the eighth row from the bottom.



- a) How many boxes are in the bottom row?
- b) Determine the general term, t_n , for the sequence.
- c) What is the number of rows of boxes in his display?

Solution using the diagram

Solution without the diagram

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Example 12: (Your Turn on p. 14)

Jonathon has been given the job of stacking cans in a similar design to that of the cereal boxes. The numbers of cans in the rows produces an arithmetic sequence. The top three rows are shown. There are 14 cans in the 8th row from the bottom and 10 cans in the 12th row from the bottom. Determine t_1 , d , and t_n for the arithmetic sequence.



Journal 1:

Amy tries to determine the 50th term of an arithmetic sequence beginning with 5 and having a common difference of 9. She calculates $(50 \times 9) + 5$ and gets an answer of 455. Is Amy correct or incorrect? Justify your answer.

Journal 2:

Determine the number of multiples of 7 between 30 and 300.

Key Ideas p. 16

Assign p. 16 - 21

#1, 2(bd), 3(c), 4(ac), 5(ac), 6(a), 7, 9, 10, 11, 13, 16, 17, 27