Mathematics 2200

Sample Midterm

Name: Key

26 Selected Response	26 marks
7 Constructed Response	30 marks
	56 Marks

FINAL

TIME: 2 HOURS

NOTE

Diagrams are not necessarily drawn to scale.

FORMULAE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

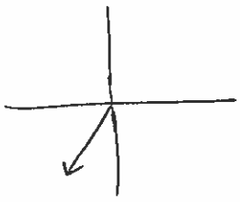
$$a^2 = b^2 + c^2 - 2bccosA$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Selected Response:

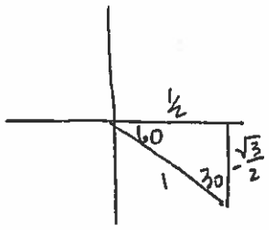
1. An angle measures 265° in standard position. What is the corresponding reference angle?

(A) 5°
(B) 15°
(C) 85°
(D) 95°

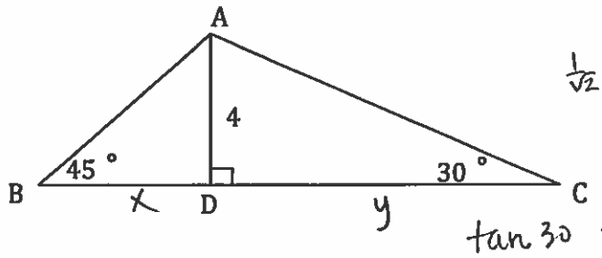


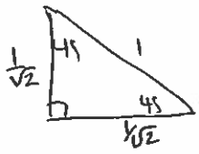
2. What is the exact value of cos (300°)?

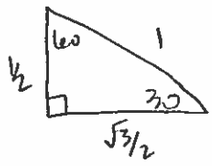
(A) $-\frac{\sqrt{3}}{2}$
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) $\frac{\sqrt{3}}{2}$



3. What is the exact length of BC?







$\frac{1}{2} \div \frac{\sqrt{3}}{2}$

$\frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

$\frac{1}{\sqrt{3}}$

$\tan 30 = \frac{4}{y}$

$y = \frac{4}{\tan 30}$

$y = \frac{4}{1/\sqrt{3}} = 4 \cdot \sqrt{3} = 4\sqrt{3}$.

$\tan 45 = \frac{4}{x}$

$x = \frac{4}{\tan 45}$

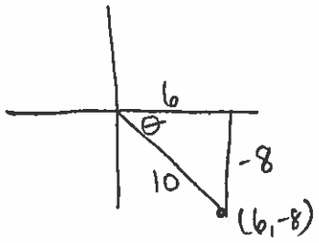
$x = \frac{4}{1}$

$x = 4$

(A) 6
(B) 12
(C) $4 + 4\sqrt{3}$
(D) $4\sqrt{2} + 4\sqrt{3}$

4. The point $(6, -8)$ lies on the terminal arm of an angle θ in standard position. What is the value of $\sin\theta$?

$6^2 + (-8)^2$

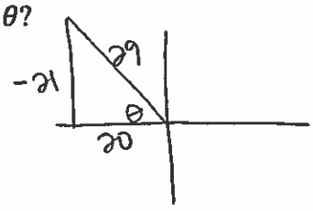


$\sin\theta = \frac{-8}{10} = \frac{-4}{5}$.

(A) $-\frac{4}{3}$
(C) $-\frac{4}{5}$
(C) $\frac{3}{5}$
(D) $\frac{4}{5}$

5. An angle θ, is plotted in standard position, and its terminal arm lies in quadrant II. Given that $\sin\theta = \frac{21}{29}$, what is $\tan\theta$?

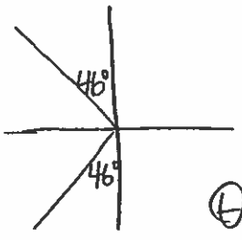
$29^2 - 21^2$



$\tan\theta = \frac{21}{20}$.

(A) $-\frac{21}{20}$
(B) $-\frac{20}{29}$
(C) $\frac{20}{29}$
(D) $\frac{21}{20}$

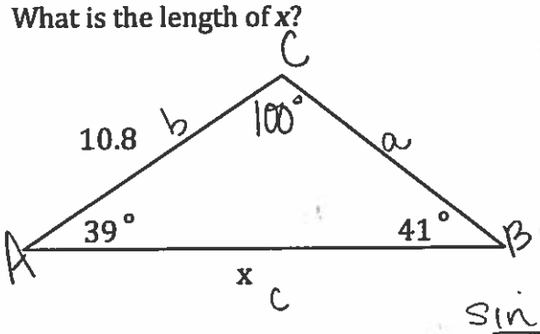
6. Solve: $\cos\theta = -0.6947$, where $0° \le \theta \le 360°$



$\theta = 134°, 226°$

(A) $\theta = 46°$ and $\theta = 134°$
(B) $\theta = 46°$ and $\theta = 314°$
(C) $\theta = 134°$ and $\theta = 226°$
(D) $\theta = 226°$ and $\theta = 314°$

7. What is the length of x?



$\frac{\sin 41}{10.8} = \frac{\sin 100}{x}$

(A) 7.2
(B) 10.4
(C) 11.3
(D) 16.2

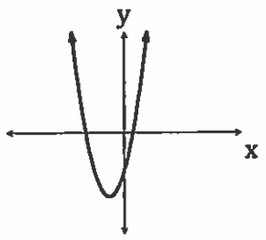
$x = 16.2$

8. Which represents the function $y = 2x^2 - 4x - 5$?

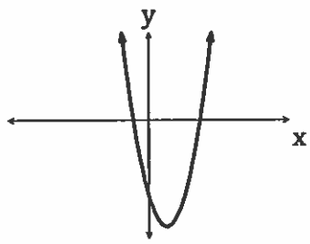
$x = \frac{-b}{2a} = \frac{4}{2(2)} = \frac{4}{4} = 1$

$y = 2(1)^2 - 4(1) - 5$
$y = 2 - 4 - 5$
$y = -7$

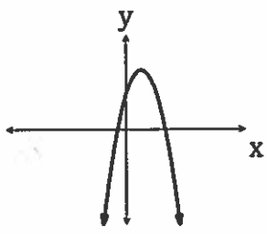
(A)



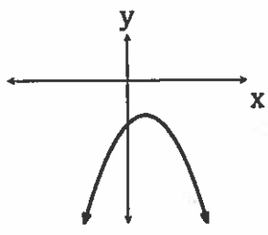
(B)



(C)



(D)



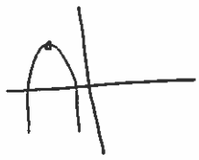
9. What is the standard form of the function $y = -3(x + 2)^2 + 4$?

(A) $-3x^2 - 8$
(B) $-3x^2 - 12x - 48$
(C) $-3x^2 - 12x - 8$
(D) $-3x^2 - 12x - 12$

$y = -3(x^2 + 4x + 4) + 4$
$y = -3x^2 - 12x - 12 + 4$
$y = -3x^2 - 12x - 8$

10. What is the range of $-3(y-5) = (x+2)^2$?

V(-2, 5)



(A) $\{y|y > -5, y \in R\}$
(B) $\{y|y \geq -5, y \in R\}$
(C) $\{y|y < 5, y \in R\}$
(D) $\{y|y \leq 5, y \in R\}$

11. What is the equation of the axis of symmetry for a parabola given by $y = -2x^2 + 12x + 5$?

$x = \frac{-b}{2a} = \frac{-12}{2(-2)} = \frac{-12}{-4} = 3$

(A) $x = 3$
(B) $x = -3$
(C) $x = -5$
(D) $x = 5$

12. Which represents a parabola with y-intercept -15 and vertex $(1, -5)$?

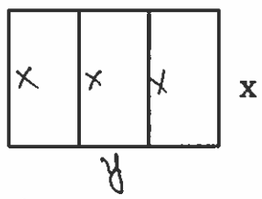
(A) $f(x) = -20(x-1)^2 - 5$
(B) $f(x) = -20(x+1)^2 + 5$
(C) $f(x) = -10(x-1)^2 - 5$
(D) $f(x) = -10(x+1)^2 + 5$

13. If $y = 2x^2 + 12x + 10$ is written in the form $y = a(x-p)^2 + q$, what is the value of q?

$y = 2(x^2+6x+9)+10-18$
$y = 2(x+3)^2 - 8$.

(A) -26
(B) -8
(C) 1
(D) 28

14. A rancher plans to use 430 m of fencing to build a cattle enclosure with three equal sections. Which represents the total area of the enclosure in terms of its width, x?



$4x + 2y = 430$.
$2y = 430 - 4x$
$y = 215 - 2x$

$A = x(215 - 2x)$

(A) $A = x(215 - 2x)$
(B) $A = x(215 - x)$
(C) $A = x(430 - 2x)$
(D) $A = x(430 - x)$

15. What are the x-intercepts of $y = x^2 + 7x - 30$?

$x^2 + 7x - 30 = 0$.
$(x+10)(x-3) = 0$
$x = -10, x = 3$

P -30
S 7
$10, -3$

(A) $(-15,0),(2,0)$
(B) $(-3,0),(10,0)$
(C) $(-2,0),(15,0)$
(D) $(-10,0),(3,0)$

16. What are the roots of $(2x-1)(x+5)=0$?

$x=\frac{1}{2}$ $x=-5$

A) $\frac{-1}{2}, -5$

B) $\frac{1}{2}, -5$

C) $\frac{1}{2}, 5$

D) $\frac{-1}{2}, 5$

17. Theresa's incorrect solution to the equation $4x^2-7x-3=0$ is shown. In which step does the **first** error occur?

Step 1 $x=\frac{7\pm\sqrt{(-7)^2-(4)(4)(-3)}}{2(4)}$

Step 2 $x=\frac{7\pm\sqrt{49-48}}{8}$

Step 3 $x=\frac{7\pm\sqrt{1}}{8}$

Step 4 $x=1, x=\frac{3}{4}$

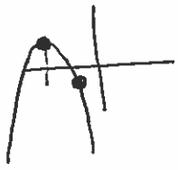
(A) 1

(B) 2

(C) 3

(D) 4

18. Which describes the quadratic function that has vertex $(-9,3)$ and passes through the point $(-4,-2)$?



(A) The axis of symmetry is $x=-9$ and the discriminant is negative.

(B) The axis of symmetry is $x=-9$ and the discriminant is positive.

(C) The axis of symmetry is $x=9$ and the discriminant is negative.

(D) The axis of symmetry is $x=9$ and the discriminant is positive.

19. Solve: $2x(x-3)+5(x-3)=0$

(A) $x=-3,\ x=-\frac{5}{2}$

(B) $x=-3,\ x=\frac{5}{2}$

(C) $x=3,\ x=-\frac{5}{2}$

(D) $x=3,\ x=\frac{5}{2}$

$2x^2-6x+5x-15=0$

$2x^2-x-15=0$

$2x^2-6x+5x-15=0$

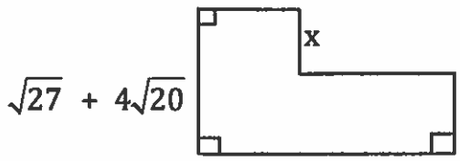
$2x(x-3)+5(x-3)=0$

$(2x+5)(x-3)=0$

$x=-\frac{5}{2}$ $x=3$

P -3 0
S -1
-6, 5

20. Determine a simplified expression for the value of x:



$(\sqrt{27} + 4\sqrt{20}) - (\sqrt{125} + \sqrt{3})$

$= 3\sqrt{3} + 4(2)\sqrt{5} - 5\sqrt{5} - \sqrt{3}$

$= 2\sqrt{3} + 3\sqrt{5}$

(A) $2\sqrt{3} + \sqrt{5}$

(B) $2\sqrt{3} + 3\sqrt{5}$

(C) $4\sqrt{3} + \sqrt{5}$

(D) $4\sqrt{3} + 3\sqrt{5}$

21. Write $4x^3y^2\sqrt{5xy}$ as an entire radical.

$\sqrt{16 \cdot 5 \cdot x^3 \cdot x^3 \cdot x \cdot y^2 \cdot y^2 \cdot y}$

$= \sqrt{80x^7y^5}$

(A) $\sqrt{20x^7y^5}$

(B) $\sqrt{20x^{10}y^5}$

(C) $\sqrt{80x^7y^5}$

(D) $\sqrt{80x^{10}y^5}$

22. Simplify completely:

$\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}} \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \frac{\sqrt{18} - \sqrt{12}}{3 - 2}$

$= \frac{3\sqrt{2} - 2\sqrt{3}}{1}$

$= 3\sqrt{2} - 2\sqrt{3}$

(A) $3\sqrt{2} - 2\sqrt{3}$

(B) $3\sqrt{2} + 2\sqrt{3}$

(C) $\frac{3\sqrt{2} - 2\sqrt{3}}{5}$

(D) $\frac{3\sqrt{2} + 2\sqrt{3}}{5}$

23. Simplify completely:

$\frac{\sqrt[3]{2}}{\sqrt[3]{6}} \cdot \frac{\sqrt[3]{6}}{\sqrt[3]{6}} \cdot \frac{\sqrt[3]{6}}{\sqrt[3]{6}}$

$= \frac{\sqrt[3]{72}}{\sqrt[3]{216}} = \frac{2\sqrt[3]{9}}{6_3} = \frac{\sqrt[3]{9}}{3}$

(A) $\frac{\sqrt[3]{3}}{3}$

(B) $\frac{\sqrt[3]{9}}{3}$

(C) $\frac{\sqrt[3]{12}}{6}$

(D) $\frac{\sqrt[3]{72}}{6}$

24. Which is $-2\sqrt[5]{160x^5y^6}$ written in simplest radical form?

(A) $-4xy\sqrt[5]{5y}$
(B) $-2xy\sqrt[5]{5y}$
(C) $-4xy\sqrt[5]{10y}$
(D) $-2xy\sqrt[5]{10y}$

$-2(2)xy\sqrt[5]{5y}$
$= -4xy\sqrt[5]{5y}$

25. Which is $\frac{\sqrt{24}}{3-\sqrt{2}}$ in simplified form?

(A) $\frac{6\sqrt{6}-4\sqrt{3}}{13}$
(B) $\frac{6\sqrt{6}-4\sqrt{3}}{11}$
(C) $\frac{6\sqrt{6}+4\sqrt{3}}{7}$
(D) $\frac{6\sqrt{6}+4\sqrt{3}}{5}$

$\frac{\sqrt{24}}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}}$
$= \frac{3\sqrt{24}+\sqrt{48}}{9-2}$
$= \frac{3(2)\sqrt{6}+4\sqrt{3}}{7}$
$= \frac{6\sqrt{6}+4\sqrt{3}}{7}$

26. Which represents the product of $\sqrt[3]{4k}\left(\sqrt[3]{12k^2}+2\sqrt[3]{8}\right)$ in simplest radical form?

(A) $2k\sqrt[3]{6}+2\sqrt[3]{4k}$
(B) $2k\sqrt[3]{6}+4\sqrt[3]{4k}$
(C) $12k\sqrt[3]{4}+2\sqrt[3]{4k}$
(D) $k\sqrt[3]{48}+2\sqrt[3]{32k}$

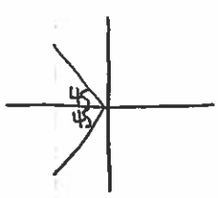
$= \sqrt[3]{48k^3} + 2\sqrt[3]{32k}$
$= 2k\sqrt[3]{6} + 2(2)\sqrt[3]{4k}$
$= 2k\sqrt[3]{6} + 4\sqrt[3]{4k}$

Constructed Response:
Answers to be written on this paper in the space provided. Show all workings.

1. Solve each of the following trigonometric equations for all possible values of θ, where $0° \leq \theta \leq 360°$. **4 marks**

(A) $\cos\theta = -\frac{\sqrt{2}}{2}$

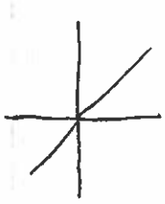
ref $\angle = 45°$



$\theta = 135°, 225°$

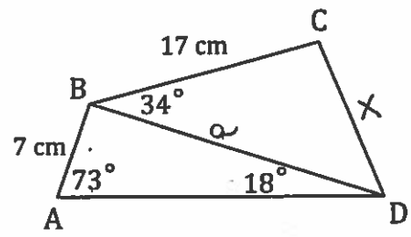
(B) $\tan\theta = \sqrt{3}$

ref $\angle = 60°$



$\theta = 60°, 240°$

2. Calculate the length of CD to the nearest tenth of a cm. 4 marks



$\frac{\sin 18}{7} = \frac{\sin 73}{a}$

$a = \frac{7(\sin 73)}{\sin 18}$

$a = 21.7\text{cm}$

$x^2 = (21.7)^2 + (17)^2 - 2(21.7)(17)\cos 34°$

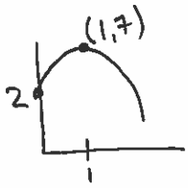
$x^2 = 470.89 + 289 - 611.66$

$x^2 = 759.89 - 611.66$

$x^2 = 148.23$

$x = 12.2\text{cm}$

3. From a height of 2 m, a volleyball is hit into the air. After 1 second, the ball reaches a maximum height of 7 m. Write the quadratic function, in the form $y = a(x-p)^2 + q$, that models the situation and use it to determine the height of the ball at 1.5 seconds. 3 marks



$y = a(x-1)^2 + 7$

$(0,2)$

$2 = a(0-1)^2 + 7$

$2 - 7 = a(-1)^2$

$-5 = 1a$

$a = -5.$

at 1.5 seconds

$y = -5(1.5-1)^2 + 7$

$y = 5.75.$

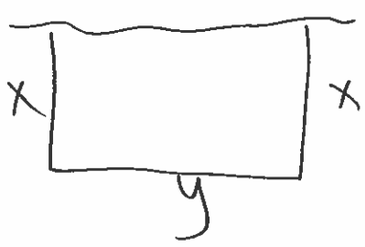
Function $y = -5(x-1)^2 + 7$

Height $y = 5.75$

4. A rectangular lot is bounded on one side by a river and on the other three sides by a total

of 100 m of fencing.

(A) Algebraically determine the quadratic function that models the area. 2 marks



$2x+y=100$

$y=100-2x$

$A=x(y)$

$A=x(100-2x)$

$A=100x-2x^2$

(B) What are the dimensions of the largest possible lot and the maximum area? 3 marks

$x=\frac{-b}{2a}=\frac{-100}{2(-2)}=\frac{-100}{-4}=25$

$y=100-2(25)$

$y=100-50$

$y=50\text{ m}$.

$A=25(50)$

$A=1250\text{ m}^2$

Dimensions 25m x 50m

5. Algebraically determine the **exact** roots, in simplest form: 4 marks

$16(x^2-1)=24(2x+1)$

$16x^2-16=48x+24$

$16x^2-48x-16-24=0$

$16x^2-48x-40=0$

$8(2x^2-6x-5)=0$

P -10
S -6

$x=\frac{6\pm\sqrt{(-6)^2-4(2)(-5)}}{2(2)}$

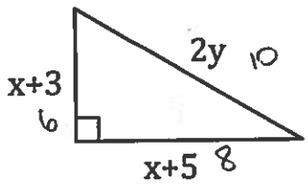
$x=\frac{6\pm\sqrt{36+40}}{4}$

$x=\frac{6\pm\sqrt{76}}{4}$

$x=\frac{6\pm2\sqrt{19}}{4}$

$x=\frac{3\pm\sqrt{19}}{2}$

6. The right triangle shown has a perimeter of 24 cm and an area of $(2y + 14)\ cm^2$. 4 marks
Algebraically determine the value(s) of x and y.



$x+3+x+5+2y=24$

$2x+2y=24-8$

$2x+2y=16$

$x+y=8$

$y=8-x$

$\frac{(x+3)(x+5)}{2}=2y+14$

$x^2+8x+15=4y+28$

$x^2+8x+15=4(8-x)+28$

$x^2+8x+15=32-4x+28$

$x^2+8x+15-60+4x=0$

$x^2+12x-45=0$

P -45
S 12

$(x+15)(x-3)=0$

$x=-15$ reject

$\boxed{x=3}$

$y=8-3$

$\boxed{y=5}$

7. Simplify each of the following.

(A) $(5\sqrt{2x}+\sqrt{5})(-4\sqrt{2x}+\sqrt{5x})$ 3 marks

$=-20(2x)+5\sqrt{10x^2}-4\sqrt{10x}+\sqrt{25x}$

$=-40x+5x\sqrt{10}-4\sqrt{10x}+5\sqrt{x}$

(B) $-3\sqrt{7r^3}\cdot 6\sqrt{7r^2}$ 3 marks

$=-18\sqrt{49r^5}$

$=-18(7)r^2\sqrt{r}$

$=-126r^2\sqrt{r}$

8. Algebraically solve $\sqrt{x-1}=x-7$.

$(\sqrt{x-1})^2=(x-7)^2$

$x-1=x^2-14x+49$

$x^2-15x+50=0$

$(x-10)(x-5)=0$

$x=10$ $x=5$ reject

Verify

$x=10$

$\sqrt{10-1}=10-7$

$\sqrt{9}=3$

$3=3$ ✓

$x=5$

$\sqrt{5-1}=5-7$

$\sqrt{4}=-2$

$2\neq -2$ x.

Solution $x=10$.