

1. Write the simplified radical form of each of the following:

$$\begin{aligned} \text{a) } \sqrt{108} & \\ &= \sqrt{36 \cdot 3} \\ &= 6\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b) } 12\sqrt{24} & \\ &= 12 \cdot \sqrt{4 \cdot 6} \\ &= 12(2)\sqrt{6} \\ &= 24\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt[3]{480} & \\ &= \sqrt[3]{8 \cdot 60} \\ &= 2\sqrt[3]{60} \end{aligned}$$

$$\begin{aligned} \text{d) } \frac{2}{3}\sqrt{63} & \\ &= \frac{2}{3}(\sqrt{9 \cdot 7}) \\ &= \frac{2}{3} \cdot 3\sqrt{7} \\ &= 2\sqrt{7} \end{aligned}$$

2. Write each as an entire radical:

$$\begin{aligned} \text{a) } 24\sqrt{6} & \\ &= \sqrt{24 \cdot 24 \cdot 6} \\ &= \sqrt{3456} \end{aligned}$$

$$\begin{aligned} \text{b) } -3\sqrt{6} & \\ &= -\sqrt{3^2 \cdot 6} \\ &= -\sqrt{9 \cdot 6} \\ &= -\sqrt{54} \end{aligned}$$

$$\begin{aligned} \text{c) } 9\sqrt[3]{7} & \\ &= \sqrt[3]{9^3 \cdot 7} \\ &= \sqrt[3]{5103} \end{aligned}$$

$$\begin{aligned} \text{d) } 5\sqrt[4]{3} & \\ &= \sqrt[4]{5^4 \cdot 3} \\ &= \sqrt[4]{1875} \end{aligned}$$

3. Simplify:

$$\begin{aligned} \text{a) } 5\sqrt{7} - 2\sqrt{7} & \\ &= 3\sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{b) } 4\sqrt{5} - 11\sqrt{5} + 3\sqrt{5} & \\ &= -7\sqrt{5} + 3\sqrt{5} \\ &= -4\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{c) } 4\sqrt{5} - 2\sqrt{75} + 3\sqrt{25} & \\ &= 4\sqrt{5} - 2\sqrt{25 \cdot 3} + 3(5) \\ &= 4\sqrt{5} - 2(5)\sqrt{3} + 15 \\ &= 4\sqrt{5} - 10\sqrt{3} + 15 \end{aligned}$$

4. Multiply and simplify:

$$\begin{aligned} \text{a) } 5\sqrt{10} \times 4\sqrt{6} & \\ &= 20\sqrt{60} \\ &= 20\sqrt{4 \cdot 15} \\ &= 20(2)\sqrt{15} \\ &= 40\sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{3}(\sqrt{11} + \sqrt{2}) & \\ &= \sqrt{33} + \sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{c) } \sqrt{8}(\sqrt{12} - \sqrt{18}) & \\ &= 2\sqrt{2}(2\sqrt{3} - 3\sqrt{2}) \\ &= 4\sqrt{6} - 6(2) \\ &= 4\sqrt{6} - 12 \end{aligned}$$

$$\begin{aligned} \text{e) } 3\sqrt{2}(2\sqrt{2} - 5\sqrt{8}) & \\ &= 6(2) - 15\sqrt{16} \\ &= 12 - 15(4) \\ &= 12 - 60 \\ &= -48 \end{aligned}$$

$$\begin{aligned} \text{f) } (3 - 2\sqrt{5})^2 & \\ &= (3 - 2\sqrt{5})(3 - 2\sqrt{5}) \\ &= 9 - 6\sqrt{5} - 6\sqrt{5} + 4(5) \\ &= 9 - 12\sqrt{5} + 20 \\ &= 29 - 12\sqrt{5} \end{aligned}$$

5. Divide. Simplify where possible.

$$\begin{aligned} \text{a) } \frac{12\sqrt{6}}{3\sqrt{2}} & \\ &= 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{b) } \frac{5\sqrt{21}}{10\sqrt{3}} & \\ &= -\frac{\sqrt{7}}{2} \end{aligned}$$

$$\begin{aligned} \text{c) } \frac{4\sqrt{27}}{\sqrt{3}} & \\ &= 4\sqrt{9} \\ &= 4(3) \\ &= 12 \end{aligned}$$

6. Identify the conjugates of the following:

$$\begin{aligned} \text{a) } 4 - 3\sqrt{5} & \\ &4 + 3\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{b) } 3\sqrt{2} + 7 & \\ &3\sqrt{2} - 7 \end{aligned}$$

$$\begin{aligned} \text{c) } -5\sqrt{8} + 3\sqrt{5} & \\ &-5\sqrt{8} - 3\sqrt{5} \end{aligned}$$

7. Rationalize the following:

$$\begin{aligned} \text{a) } & \frac{\sqrt{3}}{\sqrt{5}-\sqrt{2}} \cdot \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} \\ & = \frac{\sqrt{15}+\sqrt{6}}{5-2} \\ & = \frac{\sqrt{15}+\sqrt{6}}{3} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{\sqrt{7}+3\sqrt{2}}{9+2\sqrt{14}} \cdot \frac{9-2\sqrt{14}}{9-2\sqrt{14}} \\ & = \frac{9\sqrt{7}-2\sqrt{98}+27\sqrt{2}-6\sqrt{28}}{81-4(14)} \\ & = \frac{9\sqrt{7}-2(7)\sqrt{2}+27\sqrt{2}-6(2)\sqrt{7}}{81-56} \\ & = \frac{9\sqrt{7}-14\sqrt{2}+27\sqrt{2}-12\sqrt{7}}{25} \\ & = \frac{-3\sqrt{7}+13\sqrt{2}}{25} \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{2}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+1} \\ & = \frac{2\sqrt{2}-2}{2-1} + \frac{\sqrt{3}-1}{3-1} \\ & = \frac{2\sqrt{2}-2}{1} + \frac{\sqrt{3}-1}{2} \\ & = \frac{4\sqrt{2}-4}{2} + \frac{\sqrt{3}-1}{2} \\ & = \frac{4\sqrt{2}+\sqrt{3}-5}{2} \end{aligned}$$

8. State the restrictions.

$$\text{a) } \sqrt{x-5} \quad x \geq 5$$

$$\text{b) } \sqrt{x^3 y^8} \quad \begin{array}{l} x \geq 0 \\ y \in \mathbb{R} \end{array}$$

$$\text{c) } \frac{3}{\sqrt{x+2}} \quad x > -2$$

9. Simplify and state the restrictions.

$$\begin{aligned} \text{a) } & \frac{\sqrt{125m^3}}{5m\sqrt{5m}} \\ & = \frac{5m\sqrt{5m}}{5m\sqrt{5m}} \\ & \quad m \geq 0 \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{\sqrt{100x^3 y^6}}{10xy^3\sqrt{x}} \\ & = \frac{10xy^3\sqrt{x}}{10xy^3\sqrt{x}} \\ & \quad \begin{array}{l} x \geq 0 \\ y \in \mathbb{R} \end{array} \end{aligned}$$

$$\begin{aligned} \text{c) } & -5\sqrt{80x^4 y^7} \\ & = -5(4)x^2 y^3 \sqrt{5y} \\ & = -20x^2 y^3 \sqrt{5y} \\ & \quad \begin{array}{l} x \in \mathbb{R} \\ y \geq 0 \end{array} \end{aligned}$$

10. Multiply and write in simplified radical form.

$$\begin{aligned} \text{a) } & -3\sqrt{7r^3} \cdot 6\sqrt{7r^2} \\ & = -18r\sqrt{7r} \cdot r\sqrt{7} \\ & = -18r^2\sqrt{49r} \\ & = -18r^2 \cdot 7\sqrt{r} \\ & = -126r^2\sqrt{r} \\ & \quad r \geq 0 \end{aligned}$$

$$\begin{aligned} \text{b) } & \sqrt{6n}(7n^3 + \sqrt{12}) \\ & = 7n^3\sqrt{6n} + \sqrt{72n} \\ & = 7n^3\sqrt{6n} + 6\sqrt{2n} \\ & \quad n \geq 0 \end{aligned}$$

$$\begin{aligned} \text{c) } & (5\sqrt{2x} + \sqrt{5})(-4\sqrt{2x} + \sqrt{5x}) \\ & = -20(2x) + 5\sqrt{10x^2} - 4\sqrt{10x} + \sqrt{25x} \\ & = -40x + 5x\sqrt{10} - 4\sqrt{10x} + 5\sqrt{x} \\ & \quad x \geq 0 \end{aligned}$$

11. Divide.

$$\begin{aligned} \text{a) } & \frac{\sqrt{3x^2 y^3}}{4\sqrt{5xy^3}} \cdot \frac{\sqrt{5xy^3}}{\sqrt{5xy^3}} \\ & = \frac{\sqrt{15x^3 y^6}}{4(5xy^3)} \\ & = \frac{xy^3\sqrt{15x}}{20xy^3} \\ & = \frac{\sqrt{15x}}{20} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{3-3\sqrt{3a}}{4\sqrt{8a}} \cdot \frac{\sqrt{8a}}{\sqrt{8a}} \\ & = \frac{3\sqrt{8a} - 3\sqrt{24a^2}}{4(8a)} \\ & = \frac{3(2)\sqrt{2a} - 3(2a\sqrt{6})}{32a} \\ & = \frac{6\sqrt{2a} - 6a\sqrt{6}}{32a} \\ & = \frac{3\sqrt{2a} - 3a\sqrt{6}}{16a} \end{aligned}$$

$$\begin{aligned} \text{c) } & \frac{3n^2}{\sqrt{10n-4}} \cdot \frac{\sqrt{10n}+4}{\sqrt{10n}+4} \\ & = \frac{3n^2\sqrt{10n} + 12n^2}{10n-16} \end{aligned}$$

$$\begin{aligned} \text{d) } & \frac{4x^2-3\sqrt{3x}}{5-3\sqrt{3x^2}} \cdot \frac{5+3\sqrt{3x^2}}{5+3\sqrt{3x^2}} \\ & = \frac{20x^2 + 12x^2\sqrt{3x^2} - 15\sqrt{3x} - 9\sqrt{9x^3}}{25 - 9(3x^2)} \\ & = \frac{20x^2 + 12x^2(x)\sqrt{3} - 15\sqrt{3x} - 9(3)x\sqrt{x}}{25 - 27x^2} \\ & = \frac{20x^2 + 12x^3\sqrt{3} - 15\sqrt{3x} - 27x\sqrt{x}}{25 - 27x^2} \end{aligned}$$